

On Two Recent Approaches to Accounting for Marxian Value

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Abstract

This paper outlines two approaches to the construction of an accounting structure which relates observable prices to Marxian labour values. The first is that proposed (independently) by Duménil and Foley (D-F); the second in some sense is a generalisation and has come to be called the Temporal Single System (TSS) approach. The two approaches are based on different definitions of labour value and the paper argues that the D-F approach is more theoretically coherent and more practically useful.

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1 Introduction

The theoretical consequences of the revival of Marxist economics in the 1960s were disappointing. A large literature developed whose outcome by the mid-1970s across a wide political spectrum was agreement (albeit nuanced in a variety of ways) that the whole research programme was a degenerate one. The formalizations produced by Seton (1957) and then built upon by Morishima (1973) and by Steedman (1977) merely served to show that the labour theory of value had no operational significance in the explanation of observable phenomena. This demonstration proceeded in the following manner.

Consider the simplest case of a circulating capital model, with a uniform period of production, in which each of n industries produces a single output using a technology described by the usual \mathbf{A} matrix. The vector \mathbf{l} describes the hours of labour performed which are assumed equal to the hours of labour-power purchased, wages are paid in advance, and all labour is assumed to be productive. The equations determining the vector of labour embodied $\boldsymbol{\lambda}$ are

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}\mathbf{A} + \mathbf{l} \tag{1}$$

Thus given the technical conditions (\mathbf{A}, \mathbf{l}) and some specification of the real wage, the amounts of constant and variable capital invested in each industry are determined, constant capital being interpreted as the labour value of the means of production and variable capital as that of wage goods. Given some account of the length of the working day, surplus value and the rate of exploitation are determined. Once it is determined how capital produces surplus value, the accumulation process can then be described, to show how surplus value produces capital. In sum, the theory of capitalism is explained in terms of a class-divided system that is propelled by the drive for surplus value via the exploitation of labour.

Given constant and variable capital in labour value terms, and hence a value rate of profit R for the system as a whole, (labour value) prices of production are derived as the sum of (labour value) costs plus a (labour value) mark-up given by R . However, with this derivation, the ‘price’ of any commodity as output must be different from the ‘price’ of that same commodity as input. Since a commodity can only have one price, if the quantities of inputs are given, the value magnitudes of investment in labour-power and means of production must change. This entails that first, the procedure for determining prices of

production is incomplete, since these changes should be allowed for; and second, it is inconsistent, because such changes entail that the mark-up based on R is incorrect.

Correcting this procedure involves determining prices of production and the rate of profit simultaneously. The rate of profit r is uniquely determined by, and prices p are determined up to a scalar multiple, by

$$\mathbf{p} = (\mathbf{pA} + w\mathbf{l})(1 + r) \quad (2)$$

where w is the wage rate, and is traditionally specified in real terms as a bundle of commodities purchased by the wage. With some normalization condition serving to fix the price level, the transformation problem is concerned with how to specify a relation of determination from the ‘underlying and fundamental’ λ to the ‘surface appearances’ \mathbf{p} . This is difficult, since it is obvious from equation (1) that the proximate determinates of λ are the input-output coefficients \mathbf{A} and the labour input coefficients \mathbf{l} , and from equation (2) that those of \mathbf{p} are the same input-output and labour input coefficients plus wage data. In sum, the λ are redundant, and the relation between values and prices is nugatory.

In defensive reaction, a number of papers began to appear around the mid-1970s, which argued that this research agenda was misconceived. The general approach was to try to distinguish Marx from Ricardo, by arguing that the Ricardian labour theory of value was founded on an understanding of labour as ‘embodied labour’, whereas Marx’s notion of ‘abstract labour’ as the substance of value was not the same. These arguments took a variety of forms, largely according to how influenced they were by the rediscovery of Rubin (1972) and by how they absorbed the works of Althusser (1970) and his followers on the one hand¹, and those of Colletti (1973) on the other². While there was little consensus as to the precise difference between the Marxian and the Ricardian labour theories of value, in general two broad approaches were identified. One approach was based on a view that traditional Marxian arguments concerning the relation between values and prices had foundered through a misunderstanding of the nature of abstraction in Marx’s writings. A set of complex mediations underlay the elaboration of the abstract categories of value in their concrete referents in the empirical world, and considerable emphasis was put on an interpretation of

¹For example, Gerstein (1976).

²For example, Himmelweit and Mohun’s emphasis on abstractions as real processes, in the collection from the period edited by Mohun (1994).

the architecture of Marx's *Capital* to support this approach. An unfortunate consequence, however, was a tendency towards an almost hermetic sealing of Marxist analysis from interaction either with neoclassical economics or with the empirical world. Marxian economics became critique, with little constructive empirical analysis that was theoretically informed. An interpretation of the textual mediations in *Capital* was explicated, but this gave little clue as to how to proceed further.³

A second approach was rather different. Instead of arguing that the journey from 'capital in general' to 'many capitals' was a journey from the abstract to the concrete, it took that journey as one from the 'typical' to the individual, with notions of abstract and concrete applying as much to the 'typical' as to the individual. And it argued that a direct correspondence could be made between Marxian aggregate categories and national income accounts, and between Marxian individual categories and the accounts of capitalist firms. Such correspondences required an emphasis on the relation between value and money, a relation conspicuous by its absence in Ricardo, and alien to the barter tradition of neoclassical general equilibrium theory.

This paper focuses on this second approach. The approach is in general a descriptive one, an accounting framework for examining the historical path of capitalist development. Such a framework enables a direct focus on the measurement of values in terms of readily available national accounts data.⁴ This is important if it is believed that an approach based on class can describe the historical development of capitalism in ways in which approaches based on methodological individualism cannot. The problem is that the Marxian tradition, while certainly characterized by doctrinal controversy, has traditionally had much less to say in any rigorous sense about the empirical world. Either empirical studies are written which have no underlying theoretical basis (and hence theory is not taken seriously), or there is a concentration on theory alone (an approach that is vulnerable to an indictment of continually constructing immunising stratagems). One aim of the general approach presented here is to rectify both of these defects. But too much should not be claimed. To make the

³One example is the influential survey by Fine and Harris (1979); see also the collection edited by Fine (1986).

⁴These data are both much more widely available than input-output data, and arguably more reliable, since the input-output data themselves are derivatives of data collected in market prices in the first place.

approach explanatory or predictive would require explicit behavioural hypotheses about determination, and these are not the object of this paper. Of course, the interest in such a descriptive approach lies in the extent to which it illuminates or exemplifies the general theoretical approach presented by Marx. That said, care must be taken not to presume thereby that a descriptive approach is predictive or explanatory in the absence of specific behavioural hypotheses.

The approaches considered are ‘single system’ approaches. Instead of an equation specifying the determination of labour values, and a quite separate one specifying the determination of prices, as in equations (1) and (2), the approaches freely translate between labour value and money value magnitudes, a single system determining both. The first section outlines an approach due (in English) to Foley (1982). The second considers an approach that can be interpreted as an attempt to generalize this. The paper concludes with a discussion of how successful this attempt is.

2 The D-F approach

The sense in which money represented social labour-time was a central focus in Foley (1982), paralleling independent work (in French) by Duménil (1980). One immediate problem was that Lipietz (1982) published a survey that explicitly linked the related approaches of Duménil and Foley to the existing literature on the transformation problem, calling their approaches the ‘New Solution’. This has led to considerable confusion, because the approach proposed by Foley in particular has little if anything to do with the transformation problem. Consequently, since ‘New Solution’ is thoroughly misleading, I will call this approach, more neutrally, the D-F approach.

The process which commensurates different types of labour is the process of the sale of the output, and hence there must be some accounting relation between value and price. The most general formulation is to focus on an aggregate relationship between prices and values, in order not to presuppose any specification of individual relationships. Foley (1982) suggested that the appropriate category was ‘value added’, because this was the contribution of current labour. In the traditional circulating capital approach, the input-output equations relating the vectors of gross output \mathbf{x} and net output \mathbf{y} are

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y} \tag{3}$$

Post-multiply equation (1) by \mathbf{x} and pre-multiply equation (3) by $\boldsymbol{\lambda}$. Then,

$$\mathbf{l}\mathbf{x} = \boldsymbol{\lambda}\mathbf{y} \quad (4)$$

and the total labour performed in the production of gross output is the labour embodied in net output. This latter is total labour value added. Foley then argued that total value added should be regarded as the same aggregate, whether measured in terms of hours, as labour value added, or measured in terms of money, as money value added. As an accounting relation, their conceptual equivalence defines the value of money m (measured in hours per unit of money):

$$m = \frac{\boldsymbol{\lambda}\mathbf{y}}{\mathbf{p}\mathbf{y}} \quad (5)$$

and the inverse of the value of money is the ‘monetary equivalent of value’ (measured in units of money per hour). Equation (5) represents the basic accounting content of the labour theory of value, quite separate from, and prior to, any particular assumption made about how prices are formed, that is, whether exchange is exchange of value equivalents or of non-equivalents.

A further rationale for equation (5) could be sought in the evolution of capitalism. The process of exchange commensurates commodities according to their labour contents via a process whereby the value of any commodity i is expressed relatively in the physical form or the use-value of some other commodity j ; a social process then singles out one such equivalent to act in place of all, the ‘universal equivalent’ thereby being established as the money commodity. It is often argued that the twentieth century has seen a gradual ‘de-commodification’ of money (Foley 1983). If money is not produced as a commodity, then it has no labour content and hence has no value itself. But in an n -commodity world, money continues to have n equivalent forms, and hence the specification of the ‘value of money’ turns on an appropriate specification of that equivalent form. That is, if money has no intrinsic value itself, then the value of money is the value of something else. Formally, consideration of the ‘general form of value’ suggests immediately that, for some given stock of money, the value of any commodity will do to measure the value of money. But a normalization that privileges one particular commodity and its production conditions does not seem a satisfactory rendition of a labour theory of value, precisely because any such commodity will do. A more general approach is to use some combination of commodities as equivalent in a way that retains a labour theory of value.

One possibility is to specify all past and present labour embodied in gross output and to use the ratio of that labour value to the money value of gross output as the value of money. The difficulty with this, as Ricardo discovered, is that it is quite possible that the money value of gross output can vary with no variation in the total quantity of labour performed, if the temporal structure of embodied labour changes. As with using any particular commodity as numéraire, this does not therefore seem adequately to represent a labour theory of value. A second possibility is to restrict attention to net output and current labour performed. That is, in order to retain the link with total current labour performed, past labour should be ignored. Then the answer to the question of what money can notionally ‘command’ in exchange (for some given velocity of circulation) is total net output produced. Equation (5) formalizes this in terms of the relationship between total current labour performed and the total net output produced by that labour, that is, as the ratio of the labour value of total net output to its monetary value.

The second fundamental innovation suggested by Foley concerns the value of labour-power (*VLP*). Labour-power is an aspect or capacity of people, and is not in itself a produced commodity. Neither, except in slave societies, are people themselves produced commodities. In capitalist societies, there is therefore no labour embodied either in people or in their capacities. Hence the *VLP* cannot be defined in terms of labour-embodied, because there is none. When labour-power is sold for a wage, its value is expressed relatively, like all produced commodities, in the physical form of money, but since labour-power is not a produced commodity with any labour embodied in it, what is required theoretically is to identify those *other* social processes which determine the magnitude of the *VLP*. Functionally, the lower limit of this magnitude must be determined by the requirements of biological subsistence if social reproduction is to occur at all. But this says very little about developed capitalist economies, and the extent to which the imperatives of biological subsistence are exceeded is determined by what Marx called “a historical and moral element” (Marx 1976, p.275) and what today might be called “social norms”⁵.

Identification of these social norms and their evolution is not pursued here;

⁵Subsistence requirements “are themselves products of history, and depend therefore to a great extent on the level of civilization attained by a country; in particular they depend on the conditions in which, and consequently on the habits and expectations with which, the class of free workers has been formed”. (Marx 1976, p.275)

for this paper what is required is a unit of account for labour-power. As with money, although for different reasons, the commodity labour-power has no labour content, and again as with money an appropriate equivalent form must be specified. The usual approach specifies labour-power's equivalent form of value as the use-values purchased by the wage. The value of these use-values can then be used to fix the *VLP*. This procedure only makes sense given an assumption of equivalent exchange: the commodity labour-power, which has a value, is sold for a wage, which measures that value, and the wage is then used to purchase a bundle of commodities of equivalent value. Under these circumstances the *VLP* is the same as the value of the wage bundle of commodities. But in general this cannot be true. For each commodity in the bundle purchased by the wage is produced in its own production process, with its own technology of production, and will in general sell at a price that is not proportional to its labour-value. Hence the value of labour-power cannot be measured by the value of the commodities purchased with the wage, for that exchange is in general a non-equivalent exchange of values.

However, because labour-power is not a produced commodity, considerations of non-equivalent exchange resulting from different technologies of production do not apply to the purchase and sale of labour-power for a money wage. Hence Foley proposed that for accounting purposes the value of labour power should be understood as being measured by the wage, and left open the issue of the identification of those social processes which determine the *VLP*. More strictly, assuming some given process which converts hours of labour-power sold into hours of labour employed in production, the *VLP* per hour of labour hired is equal to the average wage rate per hour (w) multiplied by the value of money:

$$VLP = wm \tag{6}$$

And combining equations (4), (5) and (6) then shows that the *VLP* is the share of wages in money value added:

$$VLP = \frac{W}{py} \tag{7}$$

These aggregate accounting relations do not necessarily hold in each individual firm. For since the wage rate w is the average wage rate in the economy, so that

$$w = \sum_i w_i \frac{l_i x_i}{\mathbf{l}\mathbf{x}}$$

and since the VLP is formed out of a similar averaging process,

$$VLP = \sum_i VLP_i \frac{l_i x_i}{\mathbf{l}\mathbf{x}}$$

then

$$\sum_i l_i x_i (VLP_i - w_i m) = 0$$

Only in the case of a uniform wage rate will the VLP per hour of labour hired be the same in each firm, in which case variable capital in firm i at unit level is $VLP \cdot l_i$ or mwl_i .

The accounting implications are now straightforward to draw out. First, define total labour value as the sum in labour value terms of aggregate constant capital (C_L), aggregate variable capital (V_L) and aggregate surplus value (S_L):

$$\lambda \mathbf{x} = C_L + V_L + S_L \quad (8)$$

where the labour value of aggregate constant capital is the labour embodied in the means of production (the non-labour inputs):

$$C_L = \lambda \mathbf{A}\mathbf{x} \quad (9)$$

and the labour value of net output is the sum of aggregate variable capital and aggregate surplus-value, both in labour value terms:

$$V_L + S_L = \lambda \mathbf{y} \quad (10)$$

Then, since the money value of net output is the sum of aggregate wages (W) and aggregate profits (Π),

$$\mathbf{p}\mathbf{y} = w\mathbf{l}\mathbf{x} + \Pi \quad (11)$$

and since the labour value of aggregate variable capital is the VLP per hour of labour hired multiplied by the total number of hours worked, multiplying equation (6) through by $\mathbf{l}\mathbf{x}$ shows that aggregate variable capital in labour value terms is equal to total wages multiplied by the value of money:

$$V_L = Wm. \quad (12)$$

Then aggregate surplus value in labour value terms is equal to aggregate profits multiplied by the value of money, sometimes rather loosely specified as ‘total surplus value equals total profit’:

$$S_L = \Pi m \quad (13)$$

It immediately follows that if the rate of exploitation (e) is defined as the rate of surplus value S_L/V_L , that rate is the aggregate profit-wage rate in the economy:

$$e = \frac{S_L}{V_L} = \frac{\Pi}{W} \quad (14)$$

It should now be clear why it is seriously misleading to term this approach the ‘New Solution’. For the above account of the D-F approach is valid whatever prices happen to be, and whether or not there is fixed capital and joint production. Thus the D-F approach is not in itself a ‘solution’ to anything. It is rather an approach to the labour theory of value that provides an ex post accounting system that is theoretically coherent, compatible with accounting practice in capitalist society, and at least consonant with the theoretical categories developed by Marx in *Capital*. As an accounting system, no relations of determination, no relations of causality are expressed about the value of money, the *VLP* or the length of the working day. But the D-F approach does enable the construction of empirical measures, and while some theory is required to drive observation and measurement, the history of science strongly suggests that informed measurement is an important way of accumulating knowledge about precisely what it is that theory has to explain.

3 The TSS approach

In the D-F approach, the value of money is defined by a postulated equivalence between the labour value and the money value of net output. Given that definition, then the value of money exactly translates the value of labour-power per hour of labour hired into the average hourly wage rate, the labour value of aggregate variable capital into its money form of total wages, and the labour value of surplus value into its money form of total profits. This raises the question of whether a similar relation can be determined between the labour value and the money form of aggregate constant capital.

Such a relation cannot be specified in the D-F approach for constant capital, because the different means of production comprising its elements will in general be produced in processes with different compositions of capital, and hence their prices will not be proportional to their labour values. At the prevailing value of money, the labour-time equivalent of the money value of the means of production is *not* the aggregate labour value of the means of production.

But suppose it were defined to be so. It is clearly possible to define the labour value of aggregate constant capital as its money value multiplied by the value of money, and in a sense this generalizes the D-F approach by treating all inputs on the same basis. Moreover, it then enables the argument that Marx's initial 'typical' magnitudes are all denominated in money terms. This motivates the starting point of a related but different tradition from the D-F approach, initially formulated at more or less the same time, around the end of the 1970s.

Since a corollary of the proposition that all inputs should be treated symmetrically is that the labour embodied in the means of production is different from the labour value of aggregate constant capital, then the vector of values cannot be defined by equation (1). Further, the definition of value must involve some relation with money, since it is the process of commensuration of commodities in the market that renders the heterogeneous labours which produced them homogeneous. Given the starting point of a labour theory of value, so that by definition only the expenditure of labour adds value, then these considerations together suggest a formulation in which the expenditure of labour adds value to whatever is the current labour-time equivalent of the monetary value of the means of production. The monetary value of the means of production is the sum of the quantity used of each non-labour input at unit level multiplied by its current unit price. Their current labour-time equivalent is this monetary figure multiplied by the value of money. So if \mathbf{p} is the vector of prices and m^* the value of money, then the vector of TSS values $\boldsymbol{\lambda}^*$ is determined by

$$\boldsymbol{\lambda}^* = m^* \mathbf{pA} + \mathbf{l} \quad (15)$$

Equation (15) was first explicitly proposed (albeit in slightly different form) in 1982 by Wolff, Roberts and Callari (1982; 1984; see also Wolff, Callari and Roberts 1984), but it was embedded in a framework of anti-essentialist post-modernism (which arguably prejudiced its immediate acceptance). However, it was subsequently taken up by a group of theorists who ignored this original post-modernist provenance, and used it as the foundation of a wide-ranging attack on the Seton-Morishima-Steedman framework. This latter was characterized both by simultaneity and by a separate system of determination for values and for prices, as equations (1) and (2). Instead of two separate systems of determination, it was argued that there was only one – the same system which determined values also determined prices. And instead of simultaneity (a commodity having the same price whether it was input or output), it was argued that the real time

between the purchase of inputs and the sale of outputs necessitated a central focus on temporal succession. Soon dubbed a Temporal Single System (TSS) approach, the main papers are collected in Carchedi and Freeman (1996) and Freeman and Kliman (1999). I will return to the simultaneity/temporality issue later, and will concentrate first on the implications of the adoption of equation (15) as the definition of value.

Equation (15) severs any linkage between (living and dead) labour embodied on the one hand and ‘value’ on the other save through the value of money. For postmultiplying equation (15) by \mathbf{x} yields

$$\boldsymbol{\lambda}^* \mathbf{x} = m^* \mathbf{pAx} + \mathbf{lx}$$

and premultiplying equation (3) by $m^* \mathbf{p}$ yields

$$m^* \mathbf{px} = m^* \mathbf{pAx} + m^* \mathbf{py}$$

Subtracting,

$$\boldsymbol{\lambda}^* \mathbf{x} - m^* \mathbf{px} = \mathbf{lx} - m^* \mathbf{py}$$

Hence if the TSS value of money m^* is defined as the ratio of total labour value to total money value (whence by definition $\boldsymbol{\lambda}^* \mathbf{x} = m^* \mathbf{px}$ and “total value equals total price”), then $m^* = m$. Conversely, if the TSS value of money is defined in D-F terms as the equivalence of labour value added and money value added, then “total value equals total price” and $m = m^*$. Thus it is not the definition of the value of money that is critical; it is rather the definition of value by equation (15).

Total value is given by

$$\boldsymbol{\lambda}^* \mathbf{x} = C_L^* + V_L + S_L \tag{16}$$

where $C_L^* = m^* \mathbf{pAx}$ and $V_L + S_L = \mathbf{lx}$. Just as constant capital in labour value terms is the monetary value of the means of production multiplied by the TSS value of money, so it is logical to define variable capital in labour value terms as the monetary value of total wages multiplied by the TSS value of money, and surplus-value in labour value terms as the monetary value of total profits multiplied by the TSS value of money. Hence equations (12) and (13) are asserted in the TSS approach (but without the supporting reasoning of the D-F approach underlying equation (6)). Then immediately, just as constant capital

in money value terms is \mathbf{pAx} , so variable capital in money value terms is total wages, and surplus value in money value terms is total profit.

This implies that if wages are paid in advance, then, for the TSS approach, the rate of profit for the economy as a whole (R) is the same whether measured in terms of prices or labour values:

$$R = \frac{\Pi}{\mathbf{pAx} + W} = \frac{S_L}{C_L + V_L} \quad (17)$$

(whereas the second equality cannot be asserted in the same sense in the D-F approach). Again, while this holds at the aggregate level, it does not hold in general for the individual firm. For consider the i th equation in the system defined by (15):

$$\lambda_i^* = m \sum_j p_j a_{ji} + l_i.$$

Suppose that competition enforces a uniform wage rate. Then, since unit prices must cover costs and make a profit,

$$p_i = \sum_j p_j a_{ji} + wl_i + \pi_i$$

then

$$p_i = \left(\frac{\lambda_i^*}{m} - \frac{l_i}{m} \right) + wl_i + \pi_i$$

Rearranging,

$$p_i - \frac{\lambda_i^*}{m} = \pi_i - \frac{l_i}{m} (1 - wm) \quad (18)$$

so that, for any firm, the difference between its unit price and its unit value in money terms is the same as the difference between its unit profit and its appropriated surplus labour time in money terms. Hence while the aggregate rate of profit is the same whether measured in terms of prices or labour values, this is only true for individual rates of profit when individual firm profit and surplus value in money terms are the same. And in general this will not be the case.

4 Assumptions about Price Formation

Given the assumptions, both the D-F and the TSS accounts are consistent accounting frameworks for the labour theory of value, the various propositions

all holding without any particular assumption about how prices are formed. But accounts of price formation are central to accounting for the visible appearances of capitalism, and any account must bear some relation to the labour theory of value if the latter is to have any meaning. Two accounts of price formation have been important in the Marxian tradition, and each will be considered in turn.

4.1 Price Assumption 1: Prices proportional to Labour Values

Suppose that there is equivalent or equal exchange throughout the economy so that prices are proportional to values. Then for each i , the TSS equation (15) is identical to equation (1) because the distinctive TSS equation $C_L^* = m^* \mathbf{pAx}$ is identical to the D-F equation $C_L = \boldsymbol{\lambda Ax}$. Hence $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}$ and the D-F and TSS approaches are indistinguishable. Since all prices are proportional to their corresponding value magnitudes which are quantities of embodied labour, then for each firm, surplus value is equal to profit multiplied by the value of money, and it makes no difference whether individual rates of profit are measured in terms of labour values or prices. This is only to be expected, since the point of the assumption of equivalent exchange is precisely to identify the origin of exploitation and profit.

4.2 Price Assumption 2: Prices of Production

In general, because of different conditions of production, prices will not be proportional to values. Marx conceived of long-run equilibrium prices as ‘prices of production’, formed according to equation (2) out of costs marked up by the general (equalized) rate of profit. An issue for the labour theory of value then is what relation, if any, such prices bear to labour values.

4.2.1 The relation between prices and TSS values

In equation (2), the rate of profit is defined as

$$r = \frac{\Pi}{\mathbf{pAx} + W} \quad (19)$$

Since in the TSS approach each money value aggregate multiplied by m is equal to its corresponding labour value aggregate, then the Sraffian rate of profit r in equation (19) is the same as both the ‘value rate of profit’ and Marx’s general

rate of profit R in equation (17). Then for a uniform wage rate, multiplying the i th equation in (2) by m shows that the i th unit value in money terms is

$$p_i m = (c_i + v_i)(1 + r) \quad (20)$$

Now define the composition of capital used in the i th production process as

$$k_i = \frac{\sum_j p_j a_{ji} x_i}{w l_i x_i} \quad (21)$$

and the composition of capital for the economy as a whole as

$$\mathbf{k}_y = \frac{\mathbf{pAx}}{\mathbf{wlx}} \quad (22)$$

Proposition 1 (*TSS*)

Given a uniform rate of exploitation across the economy, the unit price realized for the i th commodity is greater than (equal to, less than) its value in money terms according as the composition of capital used in its production is greater than (respectively equal to, less than) the economy-wide composition of capital, both compositions being measured in terms of prices:

$$\text{sign} \left(p_i - \frac{\lambda_i^*}{m} \right) = \text{sign} (k_i - \mathbf{k}_y) \quad (23)$$

Proof. Multiply equation (18) through by $m/(c_i + v_i)$ and rearrange as

$$\frac{m\pi_i}{c_i + v_i} = \frac{m}{c_i + v_i} \left(p_i - \frac{\lambda_i^*}{m} \right) + \frac{l_i(1 - wm)}{c_i + v_i}$$

Consider each term. The left-hand side is the equalized rate of profit, and dividing through by V_L or equivalently mW and using equations (14) and (22) can be written as $e/(\mathbf{k}_y + 1)$. Similarly the second term on the right-hand side can be written as $e/(k_i + 1)$. Hence dividing the first term on the right-hand side through by v_i ,

$$\frac{e}{\mathbf{k}_y + 1} = \frac{\frac{m}{v_i} \left(p_i - \frac{\lambda_i^*}{m} \right)}{k_i + 1} + \frac{e}{k_i + 1}$$

Collecting terms

$$e \left(\frac{1}{\mathbf{k}_y + 1} - \frac{1}{k_i + 1} \right) = \frac{\frac{m}{v_i} \left(p_i - \frac{\lambda_i^*}{m} \right)}{k_i + 1}$$

so that, after simplifying,

$$p_i - \frac{\lambda_i^*}{m} = wr (k_i - \mathbf{k}_y)$$

which establishes the proposition. ■

Results like these motivate the TSS claims that Marx's own analysis is correct, and that the analyses of his critics for over a century have been wrong. These claims evidently depend upon the acceptance of the validity of the TSS value equation (15).

4.2.2 The relation between prices and D-F values

Since D-F labour values are given by λ rather than the TSS λ^* , an analogous D-F proposition to that just developed within the TSS tradition requires a different approach. Let \mathbf{u}_i be the i th unit (column) vector. Then subsystem i is defined as a system whose total net output is one unit of commodity i , and hence is \mathbf{u}_i . The gross output of subsystem i is ξ_i , determined as

$$\xi_i = \mathbf{A}\xi_i + \mathbf{u}_i$$

and hence, provided the inverse exists,⁶

$$\xi_i = [\mathbf{I} - \mathbf{A}]^{-1} \mathbf{u}_i \quad (24)$$

ξ_i is the i th column of the matrix $[\mathbf{I} - \mathbf{A}]^{-1}$, representing the quantities of all commodities directly and indirectly required by the i th industry in order to produce one unit of its output. Since l_i is the total amount of labour used directly by industry i in order to produce one unit of output of i , then the total amount of labour directly and indirectly used by subsystem i is $\mathbf{l}\xi_i = \mathbf{l}[\mathbf{I} - \mathbf{A}]^{-1} \mathbf{u}_i$. Since from equation (1), $\lambda = \mathbf{l}[\mathbf{I} - \mathbf{A}]^{-1}$, then $\mathbf{l}\xi_i = \lambda \mathbf{u}_i = \lambda_i$, where λ_i , the labour embodied in commodity i , is what Pasinetti calls the “vertically integrated labour coefficient” for commodity i (Pasinetti 1973, p.6). This notion of vertical integration can be applied to the price of production equations (2) to provide a precise measure of unequal exchange, similar but not identical to the TSS (and Marx's) rule in terms of the compositions of capital.⁷

Proposition 2 (*D-F*)

The unit price realized for the i th commodity is greater than (equal to, less than) its embodied labour value in money terms according as the vertically integrated capital (to paid) labour ratio used in its production is greater than (respectively

⁶A strong assumption, precluding consideration of fixed capital and joint production.

⁷The argument draws closely on Parys (1982).

equal to, less than) the aggregate composition of capital for the economy as a whole:

$$\text{sign}\left(p_i - \frac{\lambda_i}{m}\right) = \text{sign}\left(\mathbf{k}_i^\xi - \mathbf{k}_y\right)$$

where

$$\mathbf{k}_i^\xi = \frac{\mathbf{pA}\boldsymbol{\xi}_i}{w\lambda_i}$$

the vertically integrated ratio of capital to paid labour for subsystem i , and \mathbf{k}_y is the vertically integrated capital to paid labour ratio in the production of one unit of the composite commodity \mathbf{y} , which is just the capital to paid labour ratio for the economy as a whole given by equation (22).

Proof. Write the price of production equations (2) as

$$\mathbf{p} = r\mathbf{pA}[\mathbf{I} - \mathbf{A}]^{-1} + w(1+r)\mathbf{1}[\mathbf{I} - \mathbf{A}]^{-1} \quad (25)$$

Postmultiply by \mathbf{u}_i and rearrange to derive

$$p_i = w\lambda_i\{r\mathbf{k}_i^\xi + (1+r)\} \quad (26)$$

Then postmultiply equation (25) by the composite commodity net output \mathbf{y} and rearrange analogously to derive

$$\mathbf{py} = w\boldsymbol{\lambda}\mathbf{y}\{r\mathbf{k}_y + (1+r)\}. \quad (27)$$

Divide equation (26) by equation (27)

$$\frac{p_i}{\mathbf{py}} = \frac{\lambda_i\{r\mathbf{k}_i^\xi + (1+r)\}}{\boldsymbol{\lambda}\mathbf{y}\{r\mathbf{k}_y + (1+r)\}}$$

and hence using equation (5),

$$mp_i = \frac{\lambda_i\{r\mathbf{k}_i^\xi + (1+r)\}}{r\mathbf{k}_y + (1+r)}$$

Subtract λ_i from both sides, factorize and rearrange:

$$mp_i - \lambda_i = r\lambda_i \left(\frac{\mathbf{k}_i^\xi - \mathbf{k}_y}{r\mathbf{k}_y + (1+r)} \right)$$

Substitute for the denominator on the right-hand side from equation (27) and use the definition of m to derive

$$p_i - \frac{\lambda_i}{m} = wr\lambda_i \left(\mathbf{k}_i^\xi - \mathbf{k}_y \right)$$

which establishes the proposition. ■

5 An Evaluation

No mention has yet been made of temporal issues. The TSS approach argues strongly that temporal aspects of the production process are fundamental: inputs must be used before outputs can be produced, and hence it is conceptually wrong to presume that the input and output price for a commodity must be the same. This is obviously true in a circulating capital world for a single production process considered abstractly. On the other hand, the price at which the outputs of intermediate commodities are sold must be identical to the price at which those commodities are purchased for inputs into further production processes. So the issue is not so much the temporality of circulating capital single production processes with uniform periods of production, but the complexity of the ways in which a multiplicity of circuits of industrial capital with different periods of production intertwine. And to move outside of the circulating capital world creates ambiguities as to the very identification of intermediate inputs, because that will depend upon the degree of aggregation involved. This means that a full temporal specification of a production process is very demanding; results will inevitably depend upon the assumptions made about the degree of vertical and horizontal integration in the market, the vintage structure of the capital stock, rates of depreciation and amortization, own rates of return to each commodity, the characteristics of particular growth paths, and so on. For this reason, to move from a timeless equilibrium specification of the relationship between input and output prices to an explicitly temporal point-input point-output approach is to impose too many particular and special assumptions. The interest of the TSS contribution lies not in its advocacy of temporality, but in its assertion of equation (15).

The ‘single system’ approach of equation (15) is justified in the following way (Kliman and McGlone, 1988, p.63). Any value is always expressed as a sum of money, through the value of money. When the capitalist purchases means of production, the sum of money used represents a portion of his money-capital. That a portion of his money-capital is now physically fixed in means of production says nothing about the labour embodied in the elements of the latter. Constant capital, as a value, is not the same thing as the sum of the embodied labours of its constituent elements. And exactly the same argument holds for variable capital when ‘means of production’ is replaced with ‘labour-power’.⁸

⁸Moseley (1993) also argues that Marx’s method was to begin with these *given* aggregate

As regards variable capital, this argument is at first sight similar to that of the D-F approach, preceding equation (6) above. Yet there is a difference. Because labour-power is not a produced commodity, there is no labour embodied in it. So not only does its value (like that of all commodities) have to be expressed in something else; it is also the case that for labour-power its value *has to be the value of something else*. The only candidates for this something else are the sum of money for which labour-power is sold, and the value of the produced commodities which that sum of money then purchases. To say that the *VLP* (multiplied by the length of the working period) is the value of the commodities purchased with the wage, requires an assumption of equivalent exchange, because it is not *their* value which is being measured, but labour-power's. Then it is incoherent to use such a definition of the *VLP* in a procedure which shows that in general there cannot be equivalent exchange. So all that is left for the *VLP* is the sum of money for which it is sold, and, because labour-power is not a produced commodity, such a choice of equivalent is independent of any assumption of equivalent or non-equivalent exchange. Not only does labour-power express its value relatively in the money wage; not only is the use-value of the money wage the “mirror” (Marx 1976, p.144) for the *VLP*; the money wage (per hour, multiplied by the value of money) *is* the *VLP* (per hour of labour hired).

The situation with respect to the means of production is not symmetrical in this regard. The value of the means of production can meaningfully be expressed in both relative and equivalent forms, and, but unlike the *VLP*, means of production are produced commodities and labour is embodied in them. Hence to propose that the sum of money which purchases means of production, when multiplied by the value of money, is the value of constant capital, regardless of the embodied labour content of those means of production, is vulnerable to the criticism that such a proposition breaks the link between labour and value. Once it is asserted that total value is the value added by living labour to whatever value sum the constant capital represents at the going value of money, it is conceptually not obvious how to relate such a total value concept to the total expenditure of labour.

The same argument applies to the broader definition of the value of money. If aggregate value is to be regarded as the same aggregate, whether measured in money magnitudes of constant and variable capital.

terms of hours, as labour value, or measured in terms of money, as money value, then analogously to equation (3), the value of gross output and its price must be regarded as equivalent, their equivalence defining the value of money. But the difficulty with this approach is that value can change for reasons quite other than a change in the expenditure of labour. Changes in the temporal structure of embodiment is one such reason (recognized but unresolved by Ricardo), but there are broader ones, relating to any account of price formation. First, as mentioned above, since what is to count as an intermediate input depends upon the degree of aggregation, there is a strong element of arbitrariness in concepts such as ‘total value’, ‘total price’, ‘gross output’ and so on. Second, the market is always revaluing (in price terms) the capital stock (including inventories), and this alters through time what is to be considered as socially necessary. Consequently, there is no stability in the notion of gross concepts, and, in particular, even were ‘current gross value’ to be adequately defined, there is no necessary relation between it and current labour performed. The whole framework of a labour definition of value thereby becomes problematic.

Finally, the only point of an accounting framework is to be able to use it. A purely practical reason for preferring the D-F approach is that it avoids notorious difficulties with the valuation of the capital stock. The TSS approach requires some agreement on asset lives, depreciation and amortization rates, the valuation of stocks, and a coherent rationale for inflation accounting. All of these issues are controversial, and capital stock data are the least reliable of national accounts data. Since, further, circulating constant capital flows are netted out of national accounts data, there is an obvious instrumental reason for preferring a ‘value added approach’ to a ‘gross value’ approach. Indeed it is doubtful that a gross value approach can say anything empirical that is theoretically informed. This is not true of the D-F approach.

Hence for both theoretical and practical reasons the D-F approach is superior to the TSS approach. In the former, there is no reason ever to consider the aggregate labour value of the means of production. All accounting is in terms of money, at whatever prices happen to be. The rate of surplus-value is the aggregate profit-wage ratio, the composition of capital is the ratio of the aggregate money value of the means of production to aggregate wages, and the rate of profit is defined entirely conventionally as a ratio of money aggregates. There is no value rate of profit, either ontologically or epistemologically, and there

is nothing 'really' happening 'underneath' the price decisions of capitalists. In these terms, the D-F approach provides a theoretical basis for a coherent empirical description of capitalist economic development on the basis of the labour theory of value.

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